Is Leitgeb's Semantic Dependence a Grounding Relation?

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Background

- Recent years have seen a growing interest in a metaphysical notion of grounding [Fine, 2001, Correia, 2005, Audi, 2010, Rosen, 2010, Schaffer, 2010, Fine, 2010]
- Truth theories have frequently been defended as grounded [Herzberger, 1970, Kripke, 1975, Yablo, 1982, McCarthy, 1988, Leitgeb, 2005]
  - Avoid paradox by focusing on grounded truth.
- Working hypothesis: grounded truth an application of the general notion.

Outline

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Grounding

- Grounding relates facts, true propositions, objects, ...
- If the fact that ϕ grounds the fact that ψ, then the fact that ϕ is the metaphysical explanation as to why ψ.

  The fact that bribery is wrong is grounded in non-moral facts.

Axiomatic Approach

- I do not attempt a definition.
- Instead, I describe grounding by principles.

Syntax

- For the time being, I focus on grounding of true propositions.
- $|\phi|$ grounds $|\psi|$
- '$|\phi|$' singular term denoting the proposition that ϕ.
    - $|\text{Snow is white}|$ is the proposition that snow is white.

Explanation, Factivity, Irreflexivity

E If $|\phi|$ grounds $|\psi|$ then the truth of $|\phi|$ is the most satisfactory, ultimate explanation for $|\psi|$.

- Ultimate explanation ensures truth.
- Only truths are ultimate explanations.
- Hence,

F If $|\phi|$ grounds $|\psi|$ then ϕ, ψ.

- Nothing can explain itself, hence

I It's not the case that $|\phi|$ grounds $|\phi|$.

Transitivity

T If $|\phi|$ grounds $|\psi|$ and $|\psi|$ grounds $|\chi|$ then $|\phi|$ grounds $|\chi|$.

- If immediate grounding, then transitive concept obtained as ancestral.
  - $|\phi|$ is the mediate ground of $|\psi|$ if $|\phi|$ precedes $|\psi|$ in some series $(|\phi|_\alpha)$ where for any α, $|\phi|_\alpha$ immediately grounds $|\phi|_{\alpha+1}$.
- Transitivity and irreflexivity of grounding together imply

A If $|\phi|$ grounds $|\psi|$ then $|\psi|$ does not ground $|\phi|$.

Well-Foundedness

- Transitivity and irreflexivity require grounding chains to be non-circular.
- But grounding need not be well-founded.
- We need to distinguish between the general notion of grounding, and specific cases of groundedness.
- In special cases, we want to show that some $|\phi|$ is grounded in a given $|\psi|$.
- If there is a grounding relation that connects them, then by assumption it is well-founded.

Complete vs Partial Gdg

1. $|\phi|$ *completely* grounds $|\psi|$ iff
   - $|\phi|$ grounds $|\psi|$ and
   - $|\phi|$ sufficient for $|\psi|$.
2. $|\phi|$ *partially* grounds $|\psi|$ if
   - $|\phi|$ grounds $|\psi|$ and
   - $|\phi|$ not sufficient for $|\psi|$.

Logical Principles

- Often, grounding is assumed assumed to interact with the logical connectives according to certain principles.

G$\vee$ If ϕ then $|\phi|$ grounds $|\phi \vee \psi|$ (as does $|\psi|$).

G$\neg$ If ϕ, then $|\phi|$ grounds the truth that $\neg\neg\phi$

G$\forall$ If $\phi(t)$ then $|\phi(t)|$ grounds $|\forall x \phi(x)|$.

- I will get back to these.

Outline

Grounded Truth

- Theories of truth have frequently been defended as grounded.
- Does the relevant notion of grounding obey these principles?
- I focus on one prominent example of grounded truth theory.
- In his (2005), Hannes Leitgeb develops a *classical*, *type-free* truth theory.
- Its semantics is based on a technical concept of dependence
- Is this semantic dependence relation (the inverse of) an instance of grounding?

Leitgeb's Dependence Relation

- To the language of arithmetic add a monadic predicate 'T'.
- The extension of 'T' is a set of sentences of this very language.
- Each extension gives rise to a new model.
- The truth value of sentences ϕ containing 'T' depends on which this extension.

Definition

The truth value of ϕ depends on the set of sentences X if for all sets of sentences Y, Z $Val_Y(\phi) \neq Val_Z(\phi)$ only if $Y \cap X \neq Y \cap Z$.

A Model of Truth

- In terms of this dependence relation, Leitgeb defines a monotone operator Γ on sets of sentences,
- and proposes its fixed point Γ_{lf} as the extension of 'T'.

Definition

Leitgeb's theory of truth is the theory of the model $\mathfrak{N}_t$ which extends the standard model of arithmetic by interpreting the new predicate symbol 'T' as Γ_{lf}.

Really Grounded?

- Leitgeb calls the sentences in Γ_{lf} 'grounded' [Leitgeb, 2005, p. 168].
- But he does not connect with the metaphysical literature.

Scope of Leitgeb's Project

- Any response to the paradoxes of truth needs philosophical justification.
- Leitgeb's declared goal is to find a justified restriction (p. 156).
- His offer: Γ_{lf}. Why should we accept it?

Explanatory Character of Leitgeb's Dependence

- The sentences in Γ_{lf} are grounded in the non-semantic base theory.
- If a sentence is grounded in sentences without semantic vocabulary then it is ensured to be true or false.
- This is why according to Leitgeb Γ_{lf} is a justified restriction.
- Generally, the grounds of a sentence explain its truth value.
- To this extent, Leitgeb's dependence relation is an explanatory concept.

A Meta-Linguistic Concept

- Dependence is not expressed in the language of its relata, but in the meta-language of the semantics.
  - It cannot be – it's hyperarithmetical.
- The general grounding concept is formulated in the language of the propositions that it connects.

Undefinability Irrelevant for my Question

- Leitgeb's dependence may still be an instance of the general grounding concept.
- The principles of section 1 do not define grounding.
- If Leitgeb-dependence obeys these principles, then its undefinability motivates the axiomatic approach.

Failure of Compositionality

- Leitgeb's dependence does not obey logical principles.

G$\vee$ If $Val_{\Gamma_{lf}}(\phi) = 1$, $\phi \vee \psi$ ~~depends on~~ $\{\phi\}$.

- $\phi \vee \psi$ will not depend on $\{\phi,\psi\}$ unless ϕ and ψ contains '$T\ulcorner\phi\urcorner$' or '$T\ulcorner\psi\urcorner$'.
- Contrary to many authors (Fine, Correia, Audi), I do not think that grounding always follows logical form.

Against Logical Principles

- Fine argues for G$\vee$ as follows [Fine, 2010, p. 105]:
  1. $\phi \vee \psi$ true iff ϕ or ψ. ✓
  2. Every true complex proposition has a ground. ✓
  3. The classical truth conditions are a 'guide to ground'. ?
- Suppose (3) says

3′ If according to the truth conditions of some proposition $|\phi|$, $|\phi|$ is true if $|\psi|$ is true, then $|\psi|$ grounds $|\phi|$.

- (1) and (3) alone imply G$\vee$.
- But (3′) no more plausible than G$\vee$.
  - If we challenge G$\vee$ then we won't accept (3′).

Compositionality of Grounding and Classical Truth Conditions

G$\vee$ If $Val_{\Gamma_{lf}}(\phi) = 1$, $\phi \vee \psi$ ~~depends on~~ $\{\phi\}$.

1. $\phi \vee \psi$ true iff ϕ or ψ.

- Fine argues that G$\vee$ is the philosophical substance of (1).
- Maybe, his point is:
- Our intuitive reasons to accept the classical truth conditions are reasons to accept the logical principles of grounding.

Logical Principles are Local Principles

- This justification for G∨ only applies to proposition-grounding.
- Facts don't have truth conditions.
- Whether fact grounding is compositional depends on how we individuate facts [Correia, 2011].
  - [Snow is white or grass is blue] is grounded in [snow is white].
  - Is [water is transparent or H_2O is transparent] grounded in [H_2O is transparent].
- Logical principles hold only in some domains, but not in others.
- The non-compositionality of Leitgeb-dependence does not contradict it being a grounding notion.

Failure of Transitivity

- Leitgeb's dependence relates sentences to sets of sentences.
- Transitivity principle not even well formed:

$\mp$ If ϕ depends on X and ~~X depends on Y~~ then ϕ depends on Y.

No Quasi-Transitivity Available

- If ϕ depends on X then it also depends on any extension of X.
  - Leitgeb's definition allows for redundancies.

T′ If ϕ depends on X and $X \ni \psi$ depends on Y then ϕ depends on $X \cup Y$.

- But ϕ is not guaranteed to depend on Y alone.

Failure of Factivity

- Leitgeb-dependence connects meta-linguistic names for (sets of) sentences.

~~F.~~ If ϕ depends on X then $\not\phi$.

- Not well-formed!

Failure of Quasi-Factivity

- Idea behind F: Ground and grounded proposition are true.
- ϕ and every sentence in the set X have value 1 for some interpretation of the new predicate 'T'.
- Leitgeb's interest is in $\mathfrak{N}_t$:

Definition

$\mathfrak{N}_t$ extends the standard model of arithmetic by interpreting 'T' as Γ_{lf}.

F′ If ϕ depends on X then $\forall \psi \in X, Val_{\Gamma_{lf}}(\phi) = Val_{\Gamma_{lf}}(\psi) = 1$

- F′ expresses factivity for grounding – and it fails.
- '$T \ulcorner 0 = 1 \urcorner$' depends on $\{0 = 1\}$, but neither itself nor 0=1 are true in Leitgeb's intended model.

It’s Not that Easy…

- Leitgeb’s dependence relation does not obviously obey the general grounding principles.
- This wasn’t to be expected – they’re concepts from different research programmes.
- However, I will argue that Leitgeb’s dependence relation has a grounding core.

Outline

Principles of Grounding

Leitgeb's Semantic Dependence

The Grounding Core of Leitgeb's Dependence

The Goal

- Leitgeb's relation cannot be described as a proper grounding relation because it is
  1. not transitive
  2. not factive
- I will identify a factive and transitive concept based on Leitgeb's dependence.

Transitivity

- Quasi-transitivity failed – let's define transitive relation based on Leitgeb's dependence relation.

Definition

$\phi \Delta_0 \psi$ iff $\exists X \ni \psi$ such that ϕ depends on X.

- Possibly, X contains garbage – ψ may be irrelevant for the truth value of ϕ.

Essential Dependence

- Leitgeb offers a stricter concept.

Definition

ϕ *essentially* depends on X iff $X = \cap\{Y : \phi \text{ depends on } Y\}$

Definition

$\phi\Delta_1\psi$ iff $\exists X \ni \psi$ such that ϕ *essentially* depends on X.

- Δ_1 is an adequate variant of Leitgeb's dependence concept for sentences.

Transitivity Regained

- If ϕ *essentially* depends on X then X unique.
- Δ_1 expresses immediate dependence.
- Therefore, use the ancestral.

Definition

$\phi \Delta_2 \psi$ iff ϕ precedes ψ in a sequence (χ_α), where for every α, $\chi_\alpha \Delta_1 \chi_{\alpha+1}$.

Factivity

- Leitgeb's semantic dependence relation does not obey

F′ If ϕ depends on X then $\forall \psi \in X, Val_{\Gamma_{lf}}(\phi) = Val_{\Gamma_{lf}}(\psi) = 1$

- Simple solution: Restrict Δ_1 to Γ_{lf}.

Definition

$\phi \Delta_3 \psi$ if $\exists X \subseteq \Gamma_{lf}$ such that $\phi \in X$ ϕ *essentially* depends on X.

- Δ_3 is immediate, factive dependence.
- Trivially factive!
- Γ_{lf} is Leitgeb's definition of truth [Leitgeb, 2005, Def. 18].
- Therefore, it is legitimate to focus on this restriction.

The Grounding Core of Leitgeb's Dependence

1. Invert Δ_3
2. Take the ancestral.

Definition

$\phi G \psi$ iff ϕ precedes ψ in a sequence (χ_α), where for every α, $\chi_{\alpha+1} \Delta_3 \chi_\alpha$.

- This my candidate for the grounding core.

Irreflexivity

- G is
  - explanatory
  - transitive
  - factive
- By a result of Leitgeb's [Lemma 14,8]

I. If $\phi G \psi$ then it is not the case that $\psi G \phi$.

Future Work

- Does G do all the work needed for Leitgeb's project?
- Specify the sense in which G is a concept of explanation.
- How complex is G?
- Leitgeb has developed analogous theories of
  - Grounded abstraction
  - Grounded classes

Do these have analogous grounding cores?

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Appendix

Definition of Γ

Definition

$D^{-1}(X) = \{\phi : \phi \text{ depends on } X\}$

Definition

- $\Phi_0 = \varnothing$
- $\Phi_{\alpha+1} = D^{-1}(\Phi_\alpha)$
- $\Phi_\lambda = \bigcup_{\alpha<\lambda} \Phi_\alpha$, for λ limit

Definition

- $\Gamma_0 = \varnothing$
- $\Gamma_{\alpha+1} = \{\phi \in \Phi_{\alpha+1} : Val_{\Gamma_\alpha}(\phi) = 1\}$
- $\Gamma_\lambda = \bigcup_{\alpha<\lambda} \Gamma_\alpha$, for λ limit